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THE FORMS OF ENERGY AND THEIR TRANSFORMATIONS*

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The meaning of various model concepts, particularly those devised for space and time, or of chosen general reference systems of various kinds, of general or tetrad co-ordinate systems, and of universal and particular physical characteristic concepts having the property of covariance is expounded. Basic relations, such as force or scalar energy laws for purely mechanical phenomena or small individual volumes of matter and fields in physical processes of general form in the presence of internal and external interactions accompanied by transformation among themselves of various forms of energy, are noted.

The first and second laws of thermodynamics that generalize the principle of virtual work in classical mechanics represent the mechanical and physical laws for conceptual variational states and processes. The fundamental variational equation is derived from these laws. After the form of the considered macroscopic model has been established by specifying the defining parameters, that equation enables us to obtain expressions for internal energy, properties of external effects and mechanisms of irreversible processes, to obtain a closed system of equations, and provide a mathematical formulation of various problems for solving these. The indicated elements of concretization are always present in scientific theories, and their explicit description is suggested.

In this connection problems related to practical examples of postulating and introducing in the small concepts of energy for matter and fields are discussed and respective expressions are presented for the various forms of energy influx. In other words it is a problem of general theory of postulates or substitute assumptions, acceptable from the point of view of described reality, that are to be used in foundations of thermodynamics.

Theoretical and experimental investigations in physics and, generally, in natural sciences are based on a number of concepts — characteristics of objects and phenomena — which in many cases have the form of mathematical devices defined axiomatically or by formulas and various mathematical relationships which enable us to determine them theoretically or actually, using logical reasoning or by observation and measurements experimentally obtained, or simultaneous discussion of both.

In other words, problems are formulated, defined, and effectively solved by using devised models that must represent the main, necessary and distinctive features of things and fields, and provide the perception of processes in the surrounding universe, while at the same time serving as basis for understanding and solving the multiplicity of problems related to technology, as well as those linked with human existence. A number of such basic concepts are introduced axiomatically as basic, others are derivations or secondary, expressed by specific methods in terms of the fundamental ones /1—6/. Among those exactly defined in mathematics characteristic objects are models of space and time, the concept of mass as the measure of properties, inertia, gravity, etc. of matter.

The concepts of geometric points of space, time, and of reference systems, used for introducing the coordinates of points and time are the most fundamental of all in mechanics.

Physical space and time are considered as a four-dimensional geometric manifold of points which in Newtonian mechanics constitute a three-dimensional Euclidean space exactly defined by known postulates, and time taken as an aboslute scalar used as a variable quantity measured by clock for every point of space independently of the motion and position of the point and of the clock.

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Such model of space and time is used for describing numerous practically important phenomena, and serve as a basis for further refining of model concepts of space and time in physics. Thus in the Special Theory of Relativity (STR), whose origin is linked with the electromagnetic field theory, the three-dimensional space and time are considered as a single whole in a four-dimensional pseudo-Euclidean space. In the General Theory of Relativity (GTR) space and time are considered as a curved four-dimensional pseudo-Riemannian space which, unlike the pseudo-Euclidean space in STR, is not a priori defined but must be determined together with the phenomena that take place in the matter and fields, particularly in the presence of electromagnetic fields.

The problem of physical simulation of space and time can by no means be considered settled. Further complication and modification of these concepts is already being debated in works devoted to future problems of cognizance. For instance, four-dimensional metric affinely connected non-Riemannian spaces with torsion are considered in contemporary theories of supergravitation.

In spite of this each of the classic theories, such as the Newtonian mechanics, STR and GTR will for ever retain their importance for the physical cognition of Nature within their respective limits. This also applies to other physical models of matter and field.

The essence of model representation of space and time is the same as that of other definitions of model objects, and of problem formulations. For example, television towers or rockets may initially be considered as elastic beams or rods. However, the initial rough schemes under the effect of technological requirements and of the necessity for further knowledge become more complex. The complication of space and time models are more universal and have a particular importance in science.

It should be noted that the essence of Newtonian mechanics in the sense of its laws becomes clearer after its extension in practice beyond its limits into STR and GTR. This is particularly so in connection with the understanding of the most fundamental properties of inertia and of inertial reference systems, concepts of the nature of gravitation and of forces of inertia, with that what is the motion by inertia, the meaning of invariant indications of the spring accelerometer, all of which are independent of the initial theory or the applied reference system /23/.

It may be that hitherto in the day to day humdrum physics the main importance of GTR is in the achievement of more profound knowledge of the essence and meaning of Newtonian mechanics. The perception of this is the guiding principle in the construction of new models.

The expounded above truths may appear entirely trivial, but it is not so. At some conference one of the famous professors said: "You talk here of various models, while we study metal."

Present times are characterized by the study of numerous phenomena under more and more complex conditions, taking into account various fine details that previously were disregarded. The availability of powerful computational devices and the multiplicity of remarkable measuring instruments facilities of various kinds enable us to find answers to correctly formulated questions. In connection with this investigations have moved toward more profound theoretical analysis of problems related to simulation and with correct formulation of new problems /6,17, 18,22/. Formulation of appropriate schemes and concrete definition of mathematical problems often represents one of the main scientific achievements which in itself represents more than 60% of the path to the realization of final aims sought in practice or theory.

The theoretical and practical construction of new models of matter and fields is the most urgent problem which has a rich history of its own, but experience shows the need for ordering that activity in accordance with developed earlier successful methods of dealing with these questions, using modern achievements in physics with explicit foundation on proved universal physical laws, such as the first and second laws of thermodynamics, the properties of covariance of mathematical formulations of physical relations, the use of approved methods of allowing for the macroscopic irreversibility of phenomena, etc.

Current concepts of energy, forces, and the mechanisms of interaction between the interior of matter and fields must be defined more precisely and completely in explicit form, whenever a known clarity had been already achieved. In some cases such refinements in micro- and macroscopy represent the most important subject of modern investigations, since these questions often lack of necessary answers.

In Newtonian mechanics dynamic relations in analytical mechanics and in simplest models of continuous medium are derived from Newton's laws with the addition of related laws for acting forces, tested in experiments.

Consider the basic equation for a material point or the translational motion of a body

$$m\mathbf{a} = \mathbf{F} \tag{1}$$

where m is the mass, $\mathbf{a}$ the vector of the point acceleration relative to an inertial system of coordinates, and $\mathbf{F}$ is the resultant force vector.

Equation (1) is extended by known methods to models of systems of material points with imposed constraints.

In applications the vector equation (1) and its extensions are obviously equivalent to the conceptually introduced scalar equation for respective virtual displacements

$$\left(-m\frac{d^2x_i}{dt^2} + X_i\right)\delta x^i = 0 \tag{2}$$

where $x_i = x^i$ $(i = 1, 2, 3)$ are Cartesian coordinates, t is the time, X_i are components of the acting force $\mathbf{F}$, and δx^i the conceptually introduced components of vectors $\delta\mathbf{r}$ of virtual displacements of the mobile medium points. Here and subsequently equal covariant and contravariant indices imply summation.

It is possible to separate in formulas for the active force components X_i the potential component and write

$$X_i = -\frac{\partial U}{\partial x^i} + X_i', \quad \delta U = \frac{\partial U}{\partial x^i}\delta x^i$$

where the scalar function U is called potential energy.

We shall furthermore denote the three-dimensional scalar $mv^2/2$ by T which we shall call kinetic energy ($\mathbf{v} = d\mathbf{r}/dt$). After obvious calculations equality (2) can be expressed in the form

$$\delta(T - U) - \frac{d}{dt}(mv_i\delta x^i) + X_i'\delta x^i = 0 \tag{3}$$

For real motions with $\delta x^i = dx^i$ the equivalent equalities (1) – (3) may be rewritten in the form

$$-d(T + U) + X_i'dx^i = 0 \tag{4}$$

Equation (4) implies by definition that the total mechanical energy equal to $T + U$ is not preserved, its increment $d(T + U)$ is equal to $X_i'dx^i$ which may not be zero and which in the case of finite bodies can in many cases be interpreted as increments of other forms of energy that balance the mechanical energy change.

Extension of the energy equation (4) to the equation of the first law of thermodynamics which does not directly follow from (1) is in the form of equation

$$-d(T + U) + dQ = 0 \tag{5}$$

where dQ denotes the energy influx which in Newtonian mechanics for a material point can be attributed to the elementary work of some generalized forces, and in the mechanics of finite bodies to the flux of some other form of energy, sometimes of heat energy which is balanced by mechanical energy changes.

In the mechanics of a point the first law of thermodynamics is expressed by Eq. (5) when $dQ = 0$, i.e. when transformations of mechanical energy into other forms of energy are absent. In the presence of such transformations Eq. (5) is the law of energy conservation in thermodynamics which does not exist in mechanics for purely mechanical energy.

In the theory of continuous media dQ can have a different purely mechanical nature. For instance, it can represent the work of conventional surface forces at the body boundaries, the work of external mass forces inside the body, or the work of mass or surface forces of a higher order when internal interactions are defined by higher order tensors (e.g. a force couple, etc.). However, Eq. (5) generally represents the first law of thermodynamics in which dQ is the increment of various forms of energy that can be taken as, for instance, thermal, chemical, or electromagnetic energy.

The scalar equalities

$$\begin{aligned}\delta T - d/dt\,(mv_i\delta x^i) &= -m\mathbf{a}\delta\mathbf{r} \\ -dT &= m\mathbf{a}d\mathbf{r}\end{aligned} \tag{6}$$

represent nothing else then the elementary work of the external force of inertia on the conceptually introduced virtual displacements δx^i or on real displacement dx^i

The scalar equation

$$\delta (T - U) - d/dt\,(mv_i \delta x^i) + \delta Q = 0 \tag{7}$$

is the extended form of Eq.(3) which after summation (integration) over particles is extended to finite bodies, and which may be considered as the equation of the virtual energy conservation law that represents the fundamental postulate of physics corresponding to the first law of thermodynamics. We stress that in Eqs.(3) or (7) we have $T - U$, while in (4) and (5) appears $-(T + U)$.

Instead of the fundamental equation (1) and its extensions it is possible to use the fundamental energy equation (3) and (7), or the equivalent integral relations of the variational type. A detailed study of these questions is carried out in analytical mechanics.

The elementary work of external mass force of inertia over the virtual displacement $\delta \mathbf{r}$ of the center of a small individual particle of mass Δm can be defined in Newtonian mechanics as follows:

$$-\Delta m\,(\mathbf{a} \cdot \delta \mathbf{r}) = -\Delta m \left(\frac{d^2 \mathbf{r}}{dt^2}\,\delta \mathbf{r}\right) \tag{8}$$

where $\mathbf{a}$ is the vector of acceleration in any inertial reference system, in particular in any local proper reference system, $\mathbf{r}$ is the radius vector, and $\delta \mathbf{r}$ is the virtual change of $\mathbf{r}$.

In Newtonian kinematics vectors $\mathbf{a}$, $\mathbf{r}$, and $\delta \mathbf{r}$ can be considered in any coordinate systems. When a noninertial coordinate system, which can be considered as a system of coordinates x^i in transport motion, are hold the following formulas:

for the velocity

$$\mathbf{v} = d\mathbf{r}/dt = \mathbf{v}_{\text{mov}} + \mathbf{v}_{\text{rel}} \tag{9}$$

for the acceleration

$$\mathbf{a} = \mathbf{a}_{\text{mov}} + \mathbf{a}_{\text{rel}} + 2v^k_{\text{rel}} \frac{\partial}{\partial x^k} \mathbf{v}_{\text{mov}} \tag{10}$$

In these formulas $\mathbf{v}_{\text{mov}}$ and $\mathbf{a}_{\text{mov}}$ are the velocity and acceleration in any inertial reference system of points taken in the <u>mov</u>ing reference system (coordinate system), and $\mathbf{v}_{\text{rel}}$ and $\mathbf{a}_{\text{rel}}$ are the three-dimensional velocity and acceleration of motion <u>rel</u>ative to the moving reference system.

The last term in the right-hand side of (10) represents the generalized Coriolis acceleration with allowance for deformation of the moving reference system /27/. The vector formulas (9) and (10) can be expressed in terms of components in any coordinate system.

Equality (8) can be represented at any given point M as follows:

$$\begin{aligned} -\Delta m\,(\mathbf{a}\delta\mathbf{r}) &= \Delta m \left[\delta \frac{v^2}{2} - d\left(\mathbf{v}\,\frac{\delta \mathbf{r}}{dt}\right)\right] = \\ &\delta\left(\frac{\rho v^2}{2}\,dV_3\right) - \frac{\partial \rho\,(\mathbf{v}\delta\mathbf{r}\,dV_3)}{\partial t} - \frac{\partial \rho v^\alpha\,(\mathbf{v}\delta\mathbf{r})}{\partial x^\alpha}\,dV_3 = \\ &-\delta\left(\frac{\rho v^2}{2}\,dV_3\right) + \Delta m\left[\delta v^2 - d\left(\mathbf{v}\,\frac{\delta \mathbf{r}}{dt}\right)\right] \end{aligned} \tag{11}$$

where dV_3 is the three-dimensional individual volume, $\mathbf{v}$ is the velocity of points of the medium in a global inertial reference system. In the case of real motions the expression in brackets in the last of equalities (11) vanishes. The third intermediate expression in (11) contains a divergent term which can be integrated by parts; it was obtained using the equation of continuity of the form $\Delta m = \rho dV_3 = \text{const}$, or $\partial\rho/\partial t + \operatorname{div} \rho\mathbf{v} = 0$.

In the local proper inertial tetrad we have at each fixed point M, $\mathbf{v} = \mathbf{v}^* = 0$, hence $-\Delta m\,(\mathbf{a}\delta\mathbf{r}) = -\Delta m d\,(\mathbf{v}^*\delta\mathbf{r}^*/dt)$ and $-\Delta m\,(\mathbf{a}d\mathbf{r}^*) = 0$.

Thus in the energy equation (3) in proper reference systems of small particles the work of inertia forces and the kinetic energy of real motions vanish, but the work of inertia forces on virtual displacements is generally nonzero.

Equations of form (5) and (7) are considered in physics and chemistry as fundamental laws in investigations of numerous phenomena, including those in which motions can be generally neglected or only motions due to volume change are taken into account. For virtual states it is generally necessary to take into account in physics and chemistry the virtual work of inertia forces.

The first law of thermodynamics implies in the case of virtual or real processes that for isolated individualized systems or systems that do not interact with external objects but with allowance for the work of inertia forces, their energy, with kinetic energy taken into account, remains constant. When such isolated system interacts with other systems, its energy can only change owing to the exchange of energy between various systems.

For formulating the general macroscopic equation for the first law of thermodynamics it is necessary to clarify the nontrivial essence of the following concepts.

1) What is an individualized physical object?

2) What is the energy of physical objects, what are the various forms of energy and what are their general qualitative singularities?

3) Definition of energy from the point of view of various observers (for a system as a whole and for its separate parts).

From the point of view of analytical mechanics dealing with discreet systems of material points or invariable absolutely rigid bodies the answer to 1) is obvious.

As regards models of continuous media, the individuality of an isolated finite system as a whole is generally also obvious, but the individuality of its small parts is related to the possibility, determined by the model, of introducing Lagrangian space coordinates that individualize points of the medium.

In the final analysis an individual macroscopic small particle of a continuous medium represents a small variable volume of matter whose kinematic and physical states and physical processes are controlled by laws which by definition hold only for individual particles. In other words, individual particles are particles whose states are defined by laws for individual particles. The Lagrangian coordinates that individualize the points of a medium are determined by the specification of the velocity field of individual points at all t.

The introduction of a Lagrangian coordinate system in a medium consisting of a mixture of various substances and fields, each of which is defined in its own Lagrangian coordinates necessitates the introduction of supplementary conditions that define the characteristic physical properties of the individual particle models of the total continuum. In such cases the presence of diffusion, chemical reactions, and transformations of elementary and nuclear particles considerably complicate conventional concepts of individual bodies, used in analytical mechanics.

Indeed, if particles of matter of small variable volume are considered as a manifold of the same atoms and molecules, then in the course of short time intervals the chaotic motion of atoms and molecules and the diffusion process make the conventional concept of small particles in a continuous medium, as used in analytical mechanics, loose its meaning. It is obvious that even for a medium consisting of homogeneous atoms the question of individuality of particles of a continuous medium during finite time intervals is linked with known characteristic mechanical assumptions introduced implicitly or explicitly at the substitution of continually and continuously distributed continuous media for models of discretely distributed media.

New ideas about individual particles of a medium in mechanics of continuous media are, thus, closely related to nontrivial axiomatic extensions of microscopic concepts and laws (postulated not derived) to macroscopic concepts and laws formulated in mechanics of systems of discrete material points that, in turn, represent a far reaching simplified ideal schematization of phenomena in the surrounding Universe.

We point out in connection with the above that the effects of quantum mechanics on internal and external interactions result in further complication of concepts of individuality, in particular, in cases when macroscopic phenomena can be defined in classical mechanics in which quantum effects manifest themselves by separately defined thermodynamic concepts and functional constraints introduced from quantum mechanics.

As regards 2) it must be first of all noted that energy represents the fundamental model characteristic of physical objects, of matter and fields, which can be considered as the basic specified primary characteristic, similar to mass, in terms of which it is possible to define other characteristic physical properties, or a characteristic that can be determined and calculated using other characteristics taken as primary. It is, however, important to point out that it is possible to introduce and consider for any physical object its total energy and, in particular cases, the respective fractions of various forms of energy. (In the general theory of relativity the concept of a three-dimensional volume of a medium has generally no meaning). In all physical theories energy is the fundamental characteristic of the state and occurring processes of all physical objects. Characteristics of total energy and its fractions have a particular meaning for individualized objects that are considered in infinitely short time intervals or, when it is made possible by the meaning of the concept of an individualized object, during finite time intervals. The existence and essential meaning of the energy concept is derived for any physical object from the first law of thermodynamics that represents the universal statement postulated for individual objects in all physical models of matter and fields.

It should be added to the above that the appearance during the processing of experiments of discrepances in the energy balance can lead to the establishment of existence of individualized physical objects that are carriers of respective energies present in such experiments. In this way the particle neutrino was discovered.

Thus any physical model object has its characteristic — the energy, and this is always so, independently of whether that characteristic is used or appears explicitly in the theory or in experiments.

For any system consisting of a large number of particles and macroscopically defined of universal importance, similar to that of the second law of thermodynamics, is the macroscopic concept of entropy, associated with the qualitative singularities of thermal energy and irreversible process phenomena, which show that statistically only events oriented toward states of highest probability can be realized. Experiments and theory show that states with highest probabilities in numerous small volumes of a given system are characterised by sharp distribution peaks.

When devising macroscopic models of physical objects defined in acceptable form by the smallest number of characteristic parameters, it is possible, and generally necessary, to introduce in accordance with the first and second laws of thermodynamics, energy and entropy as specified functions of a generally small number of defining parameters. For many particular models these postulates based on experiments or some other preliminary theories may be replaced by other equivalent postulates for other characteristics in terms of which expressions for energy and entropy must or can be determined using special computation formulas or other methods.

As distinct from the concept of force in Newtonian mechanics as a vector, which has a limited universal meaning only for certain elementary mechanical systems and phenomena (that do not define heat and energy influx, associated with the work of ordinary forces related to magnetization and polarization of bodies and many other effects), the universal concept of energy for objects of any form is always a three-dimensional space scalar(*), while in the case of some complicated treatment the energy in the energy conservation law can be considered as a four-dimensional scalar in a four-dimensional space in three-dimensional coordinates and time (which is essential in STR and GTR).

The scalar concept of energy and the scalar nature of equations of the first and second laws of thermodynamics are associated with the covariance of laws of physics which is a natural extension to the case of total symmetry, local symmetry laws that can be represented by Galilean transformations in Newtonian mechanics or those of Lorentz in STR and GTR.

Let us now recall the basic forms of energy that may appear as arguments in thermodynamic functions and as separate terms in the equation of the first law of thermodyanics which is an extension of (3) used in investigations of physical phenomena in continuous media, matters and fields, in whcih external actions and internal processes accompanied by transformations of energy of one form to another can take place.

When fixing the forms of energy, their dependence on determining parameters, and establishing laws and their transformations it is necessary to proceed from data on the following forms of energy.

1) Nuclear energy develops at strong and weak interactions of elementary particles. A considerable amount of data on it already exists in physics, particularly about special possible conditions under which that energy can develop which, as a rule, is associated with mechanical motions of matter. In many physical and mechanical phenomena nuclear energy is not released and is only revealed by the presence of respective constants. There is, however, no doubt that in many most pressing mechanical and general physical problems already under consideration the energy contained in the nucleuses of atoms will be of paramount importance.

2) Electromagnetic energy is the basic characteristic of electromagnetic fields, which appears in microscopic structural formations of atoms, molecules and their combinations in various bodies, matters, in their internal interactions, and chemical compounds of organic and inorganic materials. Electromagnetic energy plays an important part in problems of propagation and absorption of various radiations, such as light and radio waves, laser emissions

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*) The three-dimensional scalar is invariant to transformation of three time-independent space coordinates; the four-dimensional scalar is invariant of transformation of general form of three space coordinates and time.

in electric technology, etc.

Many mechanical effects are obviously directly related and generated by electromagnetic effects.

3) **Chemical energy** is of electromagnetic nature, it is the energy of fuels, explosives, released or absorbed in chemical reactions in inorganic and organic chemistry, etc. Chemical energy liberation is evidently closely connected to mechanical phenomena generated by chemical processes.

4) **Mechanical energy** is basically the sum of potential and kinetic energies in relative motion linked to the work of acting forces and moments of various orders, mainly of gravitational and electromagnetic nature. The internal potential energy of continuous media also represents mechanical energy. Work and related energy that are due to force of inertia can be investigated in Newtonian mechanics.

5) **Heat energy** is the macroscopic energy of random structural states, microscopic processes, and internal motions in macroscopic bodies.

Transformations of regular macroscopic forms of energy, in particular of macroscopic forms of mechanical energy into heat energy often occurs. The latter is particularly important in irreversible processes. Properties of possible transformations of heat energy into other forms represent the second law of thermodynamics, and are used as the base for the introduction of such macroscopic properties as entropy.

6) **Gravitational energy** is in Newtonian mechanics a four-dimensional scalar that represents the energy of mutual attraction of bodies in accordance with Newton's law. This energy is attributed to bodies and is related to their relative situation. In GTR the gravitational energy is related to the Riemannian space curvature, is determined by the geometric properties of the four-dimensional space-time, and must be nonzero in vacuum, i.e. in space free of matter and electromagnetic field, but containing gravitational waves propagating at finite velocity. In the simplest models the derivation of formulas defining energy is equivalent to obtaining a four-dimensional energy-momentum tensor **P**. In that case the numerical value of energy is equal to the respective component $P_{4}^{\cdot 4}$ which is a three-dimensional scalar and which can vary under coordinate transformation dependent on the time coordinate.

For further development of the theory related to the definition of energy it is necessary to introduce concepts of the following reference systems and of respective systems of coordinates.

1. **The system of reference and of Lagrangian coordinates**, otherwise a coordinate system accompanying the separated individualized points of the continuous medium (matter or field) to which correspond uniquely defined constant values of coordinates ξ^1, ξ^2, ξ^3, while the time coordinate ξ^4 varies along nonintersecting trajectories (world lines in the four-dimensional space for each individualized point of the medium). The various accompanying systems of coordinates ξ^i and η^i for a given accompanying reference system are related by univalent transformations of the form

$$\eta^4 = f(\xi^1, \xi^2, \xi^3, \xi^4) \text{ and } \eta^\alpha = \eta^\alpha(\xi^1, \xi^2, \xi^3),\ \alpha = 1,2,3$$

2. **The observer's reference system and the corresponding system of coordinates** x^i $(i = 1,2,3,4)$. The observer's reference system must be specified and can be arbitrarily chosen. In Newtonien mechanics inertial systems are basic for observer's systems of reference, and laws of physics are usually formulated in them. It is, however, possible to use noninertial and generally deformable reference systems for this purpose. The observer's reference system can always be treated as accompanying some abstract ideal media or objects.

In one and the same space the functional relations

$$x^i = f^i(\xi^1, \xi^2, \xi^3, \xi^4),\ i = 1, 2, 3, 4$$

constitute the law of motion of a medium. Various characteristics, such as velocity, acceleration, strain and strain rate tensor, etc., which can be used as arguments for specified and unknown functions, may be introduced in connection with functions f^i.

In an accompanying reference system for individualized points the three-dimensional velocity and acceleration vectors are always zero at every point, but relative to the observer's system velocities and accelerations of individual points are nonzero when $\xi^\alpha = \text{const}$. On the other hand, individual particles may in time become deformed in the accompaning coordinate system. The accompanying reference system can evidently be inertial only in particular cases.

It is, however, generally possible to introduce locally for any point M of the medium at every instant of time an inertial observer for whom the velocity of point M is zero. The respective locally defined inertial coordinate tetrad is called proper coordinate system. In the proper reference system the velocity of point M is zero, its acceleration is nonzero, and the velocities of points infinitely close to M, although generally nonzero, are infinitely small.

The system of inertial tetrads of proper reference systems for each point of a medium, determined at every instant of time constitutes a nonholonomic set of reference systems which may be considered as an attribute of the noninertial accompanying reference system. Characteristic vectors and tensors can be investigated using the components and bases defined at every point in the proper reference system.

Physical relations between properties of phenomena, established theoretically or experimentally in local tetrads of proper reference systems, can be converted into observer's holonomic systems. Such conversion represents the solution of the basic system of navigation (*) /23,24/.

Thus the determining equations of continuous medium mechanics, particularly the thermodynamic equations, can be formulated in proper inertial reference systems in which the medium is at rest at any given point M. These equations and derived conclusions for the relative quiescence are transfered to the observer's system using navigational computations /23,24/.

Moreover the physical essence of the problem related to the nonholonomic system of proper tetrads is much deeper and directly associated with the definition of energy as a four-dimensional scalar.

Indeed, let us consider two inertial laboratories A and B equipped with the same instruments, but moving relatively to each other. Taking the Earth as the inertial system A, we imagine the second laboratory B to be set on a truck moving on perfectly straight road at some constant horizontal velocity v. In laboratory B we take a pellet at rest on the truck platform. In this case the inertial reference system attached to the platform is the proper system pellet for the pellet and it is obvious that in Newtonian mechanics the kinetic energy in system B is zero, while in STR the pellet energy is $E = m_0 c^2$, where m_0 is the mass of particle at rest and c is the speed of light.

On the other hand, the particle kinetic energy in Newtonian mechanics in reference system A attached to the Earth is nonzero and equal $m_0 v^2/2$, while in STR it is again $E = m_0 c^2$.

It can be shown that in STR the basic forms of energy can always be represented by four-dimensional invariants, while in Newtonian mechanics the kinetic energy of mass is not a four-dimensional invariant.

To eliminate this situation with the kinetic energy in equations of energy balance for a small individualized particle it is possible to consider instead of kinetic energy only the elementary work of external inertia forces that equals the kinetic energy increment and, by virtue of the vector nature of the second Newton's law, is independent of the selection of the inertial coordinate system.

The use of proper tetrads in system B is convenient because in the differentiated equations of the first and second laws of thermodynamics the unessential velocity $\mathbf{v}$ and the kinetic energy which depends on the selection of the reference system A, which, as a rule, is unrelated to processes studied in the accompanying reference system and with laws that are independent from the point of view of observer A who can be arbitrarily selected.

The preceding discussion of the part played by kinetic energy conforms to the traditional treatment of the energy equation, in which the latter can be written in the form of "equation of heat influx". The importance of the tetrad system B and of the respective form of energy equations becomes particularly clear when, besides mechanical energy, other form of energy are taken into account and expressions are established for them.

The derived above reference systems can be used for the mathematical definition of various physical concepts and of properties that can be expressed in terms of these and have kinematic, dynamic, or generally physical meaning.

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*) The basic problem of navigational calculations in Newtonian navigation was first generally formulated and solved by L.I. Tkachev in 1944. (See L.I. Tkachev, System of Inertial Orientation. Thesis for the degree of Doctor of Technical Sciences. Moscow, Moscow Power Institute, 1973).

We stress that each of the described reference systems can be used, on the one hand, as a preliminary for formulating and defining respective four-dimensional tensor characteristics and, on the other, enables us to consider characteristics of scalar and tensor nature determined by conditions established in a specific reference system. In this manner they can be used as characteristic parameters of phenomena and respective invariant formulations of various physical laws in any other coordinate systems.

Definition of physical properties, particularly of energy, in tetrad reference systems, which are subsequently considered in any other reference systems as a scalar, is most expedient for the following two reasons:

1) at any point M of the medium each tetrad is an inertial system in which the velocity of the considered point M is zero;

2) for any given medium the system of tetrads uniquely defines at any instant of time the chosen reference system. (To this should be added that laboratory experiments and their results are usually determined in a local inertial reference system).

The greatest physical importance of the above becomes apparent in the problem of determining the expression for electromagnetic energy that must appear in the equation of energies.

For a clearer exposition of the essence of this problem, let us again consider the laboratories in reference systems A and B, adding at each point of the medium respective tetrads of proper inertial platforms.

Let a conducting charged sphere be fixed on a perfect insulator thin rod attached to platform B which is movable relative to platform A, and let there be no other perturbing objects. In this experiment the measurements of the electric field by observer B shows in Newtonian mechanics or in STR simply a steady electrostatic Coulomb field that has no magnetic strenth, while the measurement of field by an inertial "stationary" observer A shows an unsteady electromagnetic field with magnetic strength. The results of measurements /of the same phenomenon/ by the same instruments in two inertial reference systems B and A thus prove to be different. The energy densities in vacuum outside the charged sphere are also different; the electromagnetic energy is defined in vacuum by the expression

$$(8\pi)^{-1}(E^2 + H^2)$$

Hence the energy density in the same volume depends on the selection of the inertial reference system, as amply shown by the described experiment. (For small three-dimensional velocities v the motions of system B relative to A that difference is very small being of order v/c). It should be added to the above that the electric and magnetic intensity vectors can only be determined in inertial reference systems which in STR can be global, and in GTR they can only be local near any point M, i.e. the tetrad proper reference systems.

We recall in this connection that an accompanying reference system is, as a rule, noninertial, and the observer's system is generally also noninertial. In GTR inertial tetrads can be introduced only locally close to any point of the medium, while the characteristics E and H of the electromagnetic field can only be determined by instruments linked to inertial reference systems. Obviously the same instruments moving relatively to each other yield different results.

It is possible to detect here some rough analogy with the indeterminacy principle in quantum mechanics according to which the result of measurements depend not only on the position of the observer but, also, on the arrangement of the experiment and on the type of instrumentation. In STR and GTR it is possible to indicate many quantities, including geometric ones for which obtained measurements depend on the selection of the moving observer. These effects, similar to the Doppler effect appear also in Newtonian mechanics.

In connection with this we have the problem of finding an inertial system that would ensure uniqueness of results of the electromagnetic field measurements, and a unique expression for energy and its increments in the electromagnetic field.

The achievement of this is connected with two basic conditions.

1) The energy-momentum tensor for the electromagnetic field must be fixed (the Minkowski tensor can be used). The question of fixing the electromagnetic field energy-momentum was fully clarified even in 1965 /8/, however, until recently numerous confused publications has appeared on this subject. Now, it seems, the correct understanding of the problem is universal.

2) The proper inertial reference system in which the medium is at rest must be used for the determination of the necessary energy fluxes at each point of the medium. Under such conditions the electromagnetic field energy becomes a four-dimensional scalar.

The preceding analysis shows that the understanding of the essence of the problem and of establishing the expression for electromagnetic energy appearing in the first law of thermodynamics are not entirely trivial.

All preceding conclusions reduce to the condition /statement/ that the medium and electromagnetic field density is represented by a four-dimensional scalar equal to the $T_{4\cdot}{}^{\cdot 4}$ component of the energy-momentum tensor defined at each point of the medium in the proper inertial reference system positioned locally.

The problem of gravitational field energy is even more complicated in GTR. For more than 70 years various ideas were proposed for defining the gravitational field energy by means of a pseudo-tensor of that field energy-momentum; however, such pseudo-tensors determined in reference systems that were not uniquely defined have no correct physical meaning. In one of his recent works the author had derived a uniquely determined existing accompanying coordinate system geometrically defined for the Riemannian space. Using this exactly and uniquely determined reference system, a physically correct real (not pseudo-) tensor of energy-momentum of gravitational field in GTR was constructed /26/. However, various variants of the numerical value of the gravitational field invariant energy are possible in this construction. Selection of a variant can be justified, simple, and confirmed experimentally, but is complicated by that the disclosure in experiments of certain effects and gravitational waves is at present associated with, so far unsurmounted experimental difficulties. The essence of this remark, which relates to any possible theories, is of general theoretical and universal importance, since in physics the correspondence of correctly devised theories (which is important in itself) of reality must always be tested and confirmed experimentally. In other words, any devised model and all numerical values of appertaining parameters must conform to experimental data.

The preceding conclusions on energy determination and all subsequent methods related to universal thermodynamic laws are to be considered as the theoretical foundations that must always be provided for, and serve, as the original basis for further progress that takes into account the accumulated experience, and reveal the meaning of new extensions in the light of already attained results.

When devising models in general and of continuous media in particular it is necessary to introduce the sought determining variable characteristics and specified constant parameters or functions, as well as the dependent on them thermodynamic functions such as internal energy U, free energy F, entropy S, or the respective laws of influx of energy of various forms, etc.

It is also necessary to use the generalized equation of the principle of virtual work or displacement, closely linked with the fundamental equations of natural sciences, viz. the first and second laws of thermodynamics, according to which the following equation must be satisfied by the individualized small volumes dV_3 of particles and fields /5–16/:

the equation of the first law of thermodynamics

$$-\delta(UdV_3) + \delta A_{M^{(e)}} + \delta A_n{}^{(e)} + \delta Q^{(e)} + \delta Q^{**} = 0 \tag{12}$$

of the second law of thermodynamics

$$-\rho\Theta\delta S dV_3 + \delta Q^{(e)} + \delta Q' = 0 \tag{13}$$

and the law of contunuity

$$dm = \rho dV_3 \text{ and } \delta\, dm = 0 \tag{14}$$

where UdV_3 is the total energy of particles, $\delta(UdV_3)$ is the virtual energy increment for conceptually introduced phenomena, $\delta A_M{}^{(e)}$ and $\delta A_n{}^{(e)}$ are the respective virtual elementary works of all external mass and surface forces which in Newtonian mechanics must contain the work of inertia forces $-dm\mathbf{a}\cdot\delta\mathbf{r}$, $\delta Q^{(e)}$ is the total virtual heat influx which may be due to distributed external mass sources and through the surface $d\Sigma$, bounding the volume $dV_4 = dV_3 dt$; δQ^{**} is the additional virtual heat energy influx that may arise due to the presence in arguments U and S of various determining parameters, such as electromagnetic properties, quantities expressed in terms of successive derivations of the law of motion, and similar essential properties of matter and field (explicit introduction in Eq.(12) of the term dQ^{**} and substantiation of its presence was made by the author in a report to the International Congress of Applied Mechanics in 1964 /5/); dQ' is the uncompensated heat whose quantity is determined by the mechanism of irreversible phenomena. In applications conventional linear constraints are often used for the determination of dQ' of the respective quantities in the Onsager theory, or in the Onsager generalized theory, for instance, of plasticity or of the electromagnetic hysteresis, nonlinear constraints are used.

Equation (14) is written on the assumption of absence of nuclear reactions in real and varied processes, hence $ddm = \delta dm = 0$. Equation (13) is written on the assumption that temperature Θ is in absolute degrees.

By integrating Eqs. (12) and (13) over the four-dimensional volumes V_4 they can be replaced by integral relations which, generally, are no longer integral laws of energy of the same kind for individualized objects as were the initial equations, since in the integration over volume V_4 the time intervals at different particles can be different, and at different instants of time volume V_4 will contain different particles.

It is clearly obvious that it is possible to derive from the universal equations (12) and (13) the following basic variational equation:

$$\delta \int_{V_4} \Lambda \, dV_4 + \delta W^* + \delta W = 0 \tag{15}$$

where V_4 is an arbitrary four-dimensional finite space-time volume.

The Lagrangian $\Lambda = -U$ (*) with the minus sign represents the total specific thermodynamic energy extended to arbitrary conceptual processes differing only little from actual processes and from the physically determined energy of the matter and field in the varied volume V_4 composed of individual particle volumes dV_4 for the corresponding instants of time.

Note that in the case of conceptual processes the quantity Λ may contain additional terms that vanish for actual processes. The variations δ are taken at constant Lagrangian coordinates ξ^1, ξ^2, ξ^3, ξ^4 and can be replaced by variations ∂ at fixed coordinates x^i in the observer's system using the operator formula

$$\delta = \partial + \delta x^i \nabla_i \tag{16}$$

The respective theory of variations that retain the same tensor nature as the varied tensor components is not trivial. It is presented in /30/.

The scalar equations (12)—(14) and derived from them Eq. (15) are written for arbitrary continuous variations of the unknown quantities. The imposes on the coefficients at variations the requirements that they must vanish for all possible feasible particular processes and motions which leads to a closed system of Euler's equations, of equations of state and supplementary conditions at strong discontinuities and at the boundaries of the unknown functions domain.

The technique of derivation and analysis of these constraints has been developed in the general theory. The condition of covariance of physical laws and equations imply that it is necessary to represent Lagrangian Λ and the corresponding to it energy U in the form of a four-dimensional invariant-scalar which generally depends on known and, mainly, on unknown functions, particularly on a number of functions that can be expressed in terms of the law of motion $x^i = f^i(\xi^k)$ and on some physically determined variables, namely components of unknown tensor parameters $\mu^A(x^k)$ $(A = 1, 2, 3, \ldots)$ /10/, on metric tensor components g_{ij}, and on constant parameters or known functions $K^B(\xi^k)$ or $K^C(x^i)$ specified in the accompanying or the observer's system. Owing to scalar properties of Λ its arguments can be defined either in the accompanying, or the observer's system, or in the tetrad system B.

When using the fundamental equation (15) an appropriate physical meaning is assigned to each of the three scalar terms, with volume V_4 arbitrary and the variations of unknown functions of that volume boundary Σ also arbitrary /11,13/.

In conformity with the fundamental equations of thermodynamics (12), (13) and (14) the additional terms δW^* and δW are introduced in the variational equation for any volume V_4.

The scalar functional δW^* is generally defined by formula

$$\delta W^* = \int_{V_4} [\rho\Theta\delta S \, dV_3 - \delta Q' + \delta A_M^{(e)} + \delta Q_M^{**}] \frac{dV_4}{dV_3} \tag{17}$$

The quantity δW^* is determined by the influx of heat $\delta Q^{(e)} = \rho\Theta\delta S dV_3 - \delta Q'$ and mass energy $\delta A_M^{(e)} + \delta Q_M^{**}$, where δQ_M^{**} are volume (mass) energy influxes due to the presence in arguments

*) Actually Λ is the density of the Lagrangian, but for simplicity and brevity we shall call it simply Lagrangian.

of U of higher order partial derivatives of unknown functions, or owing to the variation of supplementary parameters of type μ^A or K^B.

In analytical Newtonian mechanics it is necessary to introduce the term of form δW^* when external nonpotential forces and nonholonomic constraints are present. In Newtonian mechanics variation δW^* contains a term that corresponds to the work of external forces of inertia, if Λ is expressed only in terms of potential energy U.

In reversible phenomena $dQ' = 0$, while in irreversible processes $dQ' > 0$, for instance, in the presence of viscosity, electric currents, and hysteresis it is possible in some cases to set

$$\delta Q' = (\tau_i^{\,j}\, \nabla_j \delta x^i + I^k \partial A_k)\, dV_3 \tag{18}$$

where $\tau_i^{\,j}$ is the respective tensor of the viscous stress tensor type, I^k are four-dimensional components of electric current, and A_k are four-dimensional components of the electromagnetic field vector potential. The presence of term $I^k \partial A_k$ is due to the nonconservativeness of phenomena in an electromagnetic field interacting with matter (the term $I^k \partial A_k$ appeared in the expression for δQ in /29/, and its proof was given in the improved version of that paper in /32/).

Since V_4 is arbitrary, for the scalar δW defined in Eq.(15), we find that for any variations on Σ in the absence in Λ and δW^* of derivatives of unknown function of order higher than the first, the quantity δW is of the form /13,15/

$$\delta W = \int_\Sigma (P_i^{\,j} \delta x^i + M_A^{\,j} \delta \mu^A)\, n_j\, d\sigma \tag{19}$$

where n_j are components of the unit vector of the normal to surface Σ.

The components of tensors $P_i^{\,j}$ and $M_A^{\,j}$ are determined from Eq.(15).

Tensor $\mathbf{P} = P_i^{\,j} \mathbf{\partial}^i \mathbf{\partial}_j$ is the energy-momentum tensor of the medium and field. Vectors $\mathbf{\partial}^i$ and $\mathbf{\partial}_j$ are basis coordinate vectors in the observer's system; the tensor components $M_A^{\,j}$ define the supplementary tensors similar to tensor $\mathbf{P}$. The presence of $M_A^{\,j}$ is due to energy fluxes through the surface owing to the additional parameters μ^A in arguments of Λ and in the expression for δW^*.

The derived formulas for components $P_i^{\,j}$ and $M_A^{\,j}$ represent thermodynamic equations of state.

In can be shown that in the presence inside V_4 of surfaces Σ' of strong discontinuities special conditions arise on Σ' which in that case, with formula (19) valid, represent the condition of continuity of components $P_i^{\,j} n_j\,(\Sigma')$ and $M_A^{\,j} n_j\,(\Sigma')$ on Σ' /13,16,18/.

Finally, the most fundamental of all, the dynamic equations, are obtained from (15) as Euler's equations for the basic variational equation.

It can be readily shown that Euler's equations for the variational equation (15) remain unchanged when $\Lambda + \nabla_i \Omega^i$, where Ω^i are any functions of determining parameters or, generally, of any quantities, is substituted for Λ /26/.

However, such substitution alters the expressions for the system energy and for components $P_i^{\,j}$ and $M_A^{\,j}$. In connection with this, it is necessary, when devising a model, energy U and Lagrangian Λ must be fully defined, in spite of the insensitivity of basic Euler's equations to additions of the form $\nabla_i \Omega^i$ in the Lagrangian Λ. This is important for solving the problem of the gravitational field momentum tensor. This obviously applies in all cases when the model design necessitates the establishment of equations which among other things are used for formulating boundary conditions.

The scheme expounded above for designing models is based on general physical foundations with the use of all possible physical information obtained in various experiments and theories.

Thus in continuous medium mechanics theories of statistical physics are very useful. Using these theories and some very simple classical or quantum mechanics assumptions, thermodynamics functions can be calculated in the proper reference system.

Of considerable importance are data of various kinds obtained by processing a series of simplest experiments whose results can be expressed in the form of empirical formulas that are subsequently extended to more general cases, and can generally be defined in the proper reference system. Cases are known of empirical formulas becoming the basis for very profound theoretical generalizations and new physical viewpoints.

We would also point out the specially rewarding application of Eq.(15) in the developments of various kinds of theoretical models in the theories of shallow water, various kinds of thin films, plates, shells, rods, etc. /21,28,31/.

In all these examples it is possible to lower the problem dimension by specifying a probable law of distribution for the unknown functions of three variables, of respective functions of two or one variable containing internal parametric functions of a reduced number of variables, for which Euler's equations, equations of state, and corresponding boundary and other conditions are obtained.

For model problems with reduced number of independent variables it is, for instance, possible to obtain the Lagrangian by integrating the Lagrangian corresponding to a three-dimensional problem over a thin transverse dimension with respect to which the form of unknown functions is specified with an accuracy within some unknown functions.

Such theories have been already developed and the respective models substantiated using the basic variational equation (15).

Everyone of the already known models and a number of new models are based on Eq.(15) using the minimal number of physically natural and necessary assumptions in Newtonian mechanics and STR and GTR, and recently, in the formulation of more general concepts of space and time, particularly in microscopy theories of elementary particles /7,9,12,14,19,20,25,29,30/.

However, the developed general theory associated with Eq.(15), has not yet reached full development of its potentialities in known applications.

The Lagrangian can be assumed equal $-U$ in Newtonian mechanics, as well as in STR and GTR; it is also possible in Newtonian mechanics not to introduce the specific kinetic energy T in the expression for Λ. It was shown that in that case it is necessary to include in the term δW^* the work of external inertia forces besides other influxes of external energy and of energy of the type $\rho\Theta\delta S dV_3$ and $-\delta Q'$ due to irreversible phenomena.

In many cases the formula for δW^* can be written in the form

$$\delta W^* = \delta \int_{V_4} \Lambda^* dV_4 + \delta W^{*\prime} \tag{20}$$

Hence it is possible to substitute for $\Lambda = -U$ the expression

$$\Lambda = -U + \Lambda^* \tag{21}$$

with which the physical meaning of the Lagrangian as energy U with minus sign. In analytical mechanics it is assumed in conformity with (3) that

$$\Lambda = T - U \tag{22}$$

In accordance with equality (11) it is possible to set in the last term

$$\Lambda = -(T + U) \tag{23}$$

Such rearrangement of terms in the basic variational equation does not affect Euler's equations, although in such transformations the four-dimensional invariance of each term of Eq.(15) is not maintained, but that equation remains as a whole a four-dimensional invariant.

Under condition (22) the link between the variational equation (15) and the first and second laws of thermodynamics ceases to be explicit.

It can be added to this that in STR and GTR, kinetic energy does not appear in Λ in Eq.(15). In many cases of reversible adiabatic processes in which $dQ' = 0$ and $dS = 0$ it is possible to consider (22) with $\Lambda^* = T$ and $\delta W^{*\prime} = 0$.

In irreversible processes and nonconservative systems the terms $\delta W^{*\prime}$ must always be present.

In designing new models of media and particularly of fields it is common at present to use the variational "principle" of the form

$$\delta \int_{V_4^*} L\, dV_4 = 0 \tag{24}$$

where L is specified on mathematrical considerations and is not identified with the thermodynamic energy, in any case not explicitly. Only a fixed volume V_4^* is then considered, the variations of unknown functions are generally not components of tensors and are zero at the boundary Σ^* of volume V_4^* which represents the volume of the domain in which a solution is sought for specified boundary conditions on Σ^*. The basic aim is the derivation of Euler's equations.

Possibilities of obtaining equations of state and formulas for the energy-momentum tensor for specified L and $\delta W^{*\prime} = 0$, or Λ and $\delta W^* \neq 0$ using Eq.(15) are not investigated. In connection with this wrong interpretation of the essence of Euler's equations and of the concept of the energy-momentum tensor are wide spread.

It is thus possible to point out that all designs related to new models in contemporary physics in one way or another are based on integral variational equations, unfortunately without due explicit link with their thermodynamic foundations. It should also be pointed out that in each individual case the construction of models necessitates the overcoming of difficulties caused by the technically complex definition of required assumption and their interpretation.

Some very simple models of the classical type for defining motions of material media with electromagnetic field were recently investigated in detail /30/. In these models Eq.(15) is used for deriving on thermodynamically justified Λ and δW^* Euler's equations that contain all dynamic equations, including all of Maxwell's equations. Formulas are obtained in important examples for forces of interaction between electromagnetic field and matter, which depend on properties of the field and matter, and other relations.

The basic meaning of the proposed here theoretical program is in that in mechanics and physics there is essentially only a small number of universal concepts and general propositions related to model representations specifically defined by particular postulates, and that the concepts of energy and entropy associated with the first and second laws of thermodynamics may be considered as the foundations of physics and mechanics. These foundations are few and simple; however their understanding and their creative application requires a deep penetration of the essence of scientific methods of cognizance of the surrounding Nature.

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